NORTHWESTERN
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Probability Revision

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Considers the pretest (or prior) and post test (or posterior) probability factors involved in probability revision to determine the chances of the test subject having disease. It is an extension of the analysis used in a 2x2 table.

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Problem Description

Assume that the incidence for Lyme disease in the state of Connecticut is 78 cases per 100,000. A diagnostic test for the disease has a sensitivity of 81% and a specificity of 96%. Select two different probability revision techniques and use both to calculate:

Definitions (Hunink & Glasziou, 2009)

- **True Positive Rate (TPR)** – Probability of a positive test result given that the disease is present. Also known as the *Sensitivity.*
- **True Negative Rate (TNR)** – Proportion of patients without the disease having a negative test result. Also known as the *Specificity*.
- **False Negative Rate (FNR)** – Proportion of patients with disease who have a negative test result.
- **False Positive Rate (FPR)** – Proportion of patients without the disease who have a positive test result.
- **Pretest probability –** Disease prevalence in population in a randomly chosen patient.

Assumptions and Givens

- Sn (Sensitivity or TPR) – 80%
- Sp (Specificity or TNR) – 96%

Solution Technique 1 (2x2) – Hunink pg 136-138

With the above information given we can assume that there are a total of 78 cases tests indicating disease, out of a possible 100,000 (either Truly Positive or Falsely Negative). Therefore, 100,000 – 78 equals 99,992 testing without the disease (either Falsely Positive or Truly Negative). The Pre-Test Probability or Prevalence is .078%

2 x 2 Table

		Disease (78)		No Disease (99,922)	100,000
+ (4,060)	TP	63	FP	3,997	
- (95,940)	FN	15	TN	95,925	

100,000

Sensitivity (Sn)	= 0.81 (TP / (TP + FN)
Specificity (Sp)	= 0.96 (TN / (TN + FP)

* Green highlight indicates given variables.

TP	= P (C/D) = .81 x 78 = 63
FN	= P(-C/D) = .19 x 78 = 15

TN	= P(-C/-D) = .96 x 99,922 = 95,925
FP	= P(C/-D) = .04 x 99,992 = 3,997

Formulas and Calculations

PPV	= TP / (TP + FP) = 63/4,060 = .015 or 1.5%
NPV	= TN / (TN + FN) = 95,925 / 95,940 = .99984

Test Efficiency	= (Sn + Sp / 2) = (1.77/2) = 88.5%
Prevalence	= (TP + FN) / (TP + FN + FP + TN) = 78/100,000 = .00078 = .078%

FNR	= 1 - Sn = 1 - .81 = .19 of 19%
FPR	= 1 - Sp = 1 - .96 = .04 or 4%

PLR	= Sn/ (1 - Sp) = 0.81 /.04 = 20.25
NLR	= 1-Sn / Sp = .19/.96 = .1979

Pre-Test Probability	= Prevalence = .078%
Pre-Test Odds	= Pre-Test Prob / (1 - Pre-Test Prob) = 0.078 / .922 = .0845
Post-Test Odds	= Pre-test Odds x Likelihood Ratio = .0845 x 20.25 = 1.7111
Post-Test Probability	= Post-Test Odds / (1 + Post-Test odds) = 1.711 / 2.711 = .6311

Inference

a) The probability that a person being tested in Connecticut has Lyme disease given a positive result for the test is equal to the PPV or 1.55%

b) The probability that the person has Lyme disease given a negative test result is equal to (D+|T-) or 1 – NPV = .015%.

Solution Technique 2 (Bayes Formula) – Hunink pg 140-149

The Bayes' forumula for dichotomous (+ or -) test is what we'd use to figure out the probabilities for two disease states. That equation for positive is:

$$\frac{Sensitivity\ x\ pretest\ probability}{(Sensitivity\ x\ pretest\ probability + (1 - Specificity) x\ (1 - pretest\ probability))}$$

$$P(D + | T+) = \frac{\left(P\left(\frac{T+}{D+}\middle|\right) * P(D+)\right)}{P\left(\frac{T+}{D+}\right) * P(D+) + P\left(\frac{T+}{D-}\right) * \ P(D-)} = \frac{(.81\ x\ .00078)}{(.81\ x\ .00078) + (.04)\ x\ (1 - .00078)} = \frac{.0006318}{.0406001} =$$

.0155 ← PPV

The negative test would be

$$\frac{(1 - Sensitivity)\ x\ pretest\ probability}{((1 - Sensitivity)\ x\ pretest\ probability + (Specificity)\ x\ (1 - pretest\ probability))}$$

$$P(D + | T-) = \frac{\left(P\left(\frac{T-}{D+}\middle|\right) * P(D+)\right)}{P\left(\frac{T-}{D+}\right) * P(D+) + P\left(\frac{T-}{D-}\right) * \ P(D-)} = \frac{((1 - .81)\ x\ .00078)}{((1 - .81)\ x\ .00078) + (.96)\ x\ (1 - .00078)} =$$

$$\frac{.19\ x\ .00078}{(.19\ x\ .00078) + .96\ x\ (1 - .00078)} = \frac{.0001482}{.9593992} =$$

.015 ← PPV

Inference

a) The probability that a person being tested in Connecticut has Lyme disease given a positive result for the test is equal to the PPV (D+|T+) or 1.55%
b) The probability that the person has Lyme disease given a negative test result is equal to the (D+|T-) or 0.015%. Also equivalent to 1-NPV.

Works Cited

Hunink, M., & Glasziou, P. (2009). *Decision making in health and medicine: Integrating evidence and values.* Cambridge, England: Cambridge University Press.